

Voltage-Controlled Negative Refractive Index in Vertically Coupled Quantum Dot Systems

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We demonstrate that voltage-controlled negative refractive index can be obtained in self-organized InAs quantum dot systems. As the bias voltage is changed, the refractive index can be adjusted and controlled continuously from negative to positive and simultaneously the loss of light in the system will be small. The single-negative index materials and the double-negative index materials can be achieved only by simply applying the external bias voltages. © 2008 Optical Society of America

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Far from a material with a negative index of refraction suggested by Veselago in 1968 [1], the unique physical and good optical properties such as the development of a "perfect lens" [2] in which imaging resolution is not limited by electromagnetic wavelength have attracted lots of interest. Early experiments [3,4] which achieved the negative index are some special structures with circuits and coils at microwave frequencies. Recently, some nanoscale periodic structures including photonic metamaterials demonstrate the negative index effect [5]. Khoo [6] et al. have shown that aligned nematic liquid crystal cells containing core-shell nanospheres are possible to devise a new type of metamaterial with negative index. Valentine et al. [7] experimentally realized the three-dimensional optical metamaterial with a negative refractive index. At the same time, negative index materials (NIM) are studied theoretically by many researchers [8,9] in atomic systems. Recently, properties of quantum dots (QD) in the optical field are studied [10,11]. In this letter, we propose a scheme consisting of a typical InAs self-assembled quantum dot system and show that negative index of refraction becomes accessible only simply by changing the bias voltage. The refractive index can be adjusted continuously from negative to positive as the bias voltage alters, simultaneously the loss of light in the system is very small. In such a way the single-negative index materials

(SNIM) and the double-negative index materials (DNIM) can be easily achieved in different bias voltages.

A vertically coupled InAs/InGaAs asymmetrical quantum dot system consisting of two layers of dots (the upper layer and the lower layer) with different band structures is shown in Fig.1(a). Samples are arrays of InAs dots in a matrix of InGaAs which are vertically stacked and electrically coupled in the growth direction. Dots in two different layers show a strong tendency to align vertically. The substrates and the top surface are composed with transparent and conductive materials. V is a bias voltage. The coupling of two layers QD is mainly determined by the separation distance of two layers. Fig.1(b) is a TEM picture of such self-assembled quantum dots [12]. In this quantum dot system, the lower QD is slightly small, so its energy difference between ground state and first excited state is larger than that of the upper one. From Ref. [13], we know that for QD separation $d > 9$ nm the tunneling coupling between the two dots is weak and the QD system can be discussed in terms of a simplified single-particle picture [10, 14]. Applying an electromagnetic field we can excite one electron from the valence to the conduction band in the lower dot which can in turn tunnel to the upper dot. Fig.1(c) shows this transport. Using this configuration the Hamiltonian of the system in the electric dipole approximation reads as follows [15]

$$H = \sum_{i=1}^3 \hbar \omega_i |i\rangle\langle i| - \frac{\hbar}{2} (\Omega e^{-i\nu t} |1\rangle\langle 2| + \Omega^* e^{i\nu t} |2\rangle\langle 1|) + \hbar T_c (|1\rangle\langle 3| + |3\rangle\langle 1|), \quad (1)$$

where $\hbar \omega_i$ is the energy of level i of the quantum dots, and $|i\rangle\langle j|$ ($i, j = 1, 2, 3$) are projection operators. ν is the frequency of the laser. Inter-dot tunneling between two layers is described by a single, real parameter T_c conventionally [16]. The Rabi frequency associated with the optical transition ($|2\rangle \rightarrow |1\rangle$) is $\Omega = \vec{d}_{12} \cdot \vec{E} / \hbar$ where \vec{E} stands for the complex amplitude of the positive frequency component electric field of the laser and $\vec{d}_{12} = e \langle 1 | \vec{r} | 2 \rangle$ is the electric dipole operator.

The density matrix elements of the system evolve according to the Liouville equation [17],

$$\frac{d\rho}{dt} = -\frac{i}{\hbar} [H, \rho] - \frac{1}{2} \{\Gamma, \rho\}. \quad (2)$$

In what follows we assume a diagonal relaxation matrix $\langle i | \Gamma | j \rangle = \gamma_i \delta_{ij}$, and choose the relaxation rates for off-diagonal element of the density matrix as $2\gamma_{ij} = \gamma_i + \gamma_j$, and then

$$\dot{\rho}_{12} = -(i\omega_{12} + \gamma_{12})\rho_{12} + \frac{i}{2}\Omega e^{-i\nu t}(\rho_{11} - \rho_{22}) - iT_c\rho_{32}, \quad (3)$$

$$\dot{\rho}_{32} = -(i\omega_{32} + \gamma_{32})\rho_{32} + \frac{i}{2}\Omega e^{-i\nu t}\rho_{31} - iT_c\rho_{12}. \quad (4)$$

As the lower quantum dot is initially in the ground state $|2\rangle$,

$$\rho_{22}^0 = 1, \quad \rho_{11}^0 = \rho_{33}^0 = \rho_{13}^0 = 0. \quad (5)$$

On substituting these initial conditions into (3) and (4), and making the substitutions,

$$\rho_{12} = \tilde{\rho}_{12}e^{-i\nu t}, \quad \rho_{32} = \tilde{\rho}_{32}e^{-i\nu t}. \quad (6)$$

After the substitution, this set of equation can be solved, for example, by first writing in the matrix form [17],

$$\dot{\mathbf{R}} = -\mathbf{M}\mathbf{R} + \mathbf{A} \quad (7)$$

with

$$\mathbf{R} = \begin{pmatrix} \tilde{\rho}_{12} \\ \tilde{\rho}_{32} \end{pmatrix},$$

$$\mathbf{M} = \begin{pmatrix} \gamma_{12} + i\Delta & iT_c \\ iT_c & \gamma_{32} + i\delta \end{pmatrix},$$

$$\mathbf{A} = \begin{pmatrix} \frac{i}{2}\Omega \\ 0 \end{pmatrix},$$

where $\Delta = \omega_{21} - \nu$ and $\delta = \omega_{32} - \nu = \Delta - \omega_{31}$. Then integrating

$$R(t) = \int_{-\infty}^t e^{-M(t-t')} A dt' = M^{-1} A. \quad (8)$$

We yield

$$\tilde{\rho}_{12} = \frac{\frac{i}{2}\Omega(\gamma_{32} + i\delta)}{(\gamma_{12} + i\Delta)(\gamma_{32} + i\delta) + T_c^2}, \quad (9)$$

and

$$\tilde{\rho}_{32} = \frac{\frac{i}{2}\Omega(-iT_c)}{(\gamma_{12} + i\Delta)(\gamma_{32} + i\delta) + T_c^2}. \quad (10)$$

From Eq.(9) and Eq.(10), we have

$$\tilde{\rho}_{32} = -\frac{iT_c}{(\gamma_{32} + i\delta)}\tilde{\rho}_{12}. \quad (11)$$

Therefore, the electric polarizability χ_e is given by

$$\chi_e(\omega) = \frac{2Nd_{12}\tilde{\rho}_{12}}{\varepsilon_0 E} = \frac{iNd_{12}^2}{\varepsilon_0 \hbar} \frac{\gamma_{32} + i\delta}{(\gamma_{12} + i\Delta)(\gamma_{32} + i\delta) + T_c^2}, \quad (12)$$

where N is the density of the double quantum dots. The magnetic susceptibility χ_m is expressed as

$$\chi_m(\omega) = \frac{2Nm_{32}\tilde{\rho}_{32}}{H_m} = -\frac{m_{32}}{d_{12}c} \sqrt{\frac{\mu_r}{\varepsilon_r}} \frac{iT_c}{(\gamma_{32} + i\delta)} \chi_e(\omega), \quad (13)$$

where m_{32} is the magnetic dipole matrix element and c is the speed of light in vacuum. In Eq.(13), we have used the relation $H_m = \sqrt{\frac{\varepsilon_r \varepsilon_0}{\mu_r \mu_0}} E$ between the envelopes of magnetic and electric fields, where $\varepsilon_r = 1 + \chi_e$ and $\mu_r = 1 + \chi_m$, and then we have

$$\epsilon_r = 1 + \chi_e = 1 + \frac{iNd_{12}^2}{\varepsilon_0 \hbar} \frac{\gamma_{32} + i\delta}{(\gamma_{12} + i\Delta)(\gamma_{32} + i\delta) + T_c^2}, \quad (14)$$

$$\mu_r = 1 + \frac{\eta^2 \pm \sqrt{\eta^4 + 4\eta^2}}{2}, \quad (15)$$

where

$$\eta = -\frac{m_{32}}{d_{12}c} \frac{iT_c}{(\gamma_{32} + i\delta)} \frac{\chi_e}{\sqrt{1 + \chi_e}}. \quad (16)$$

The results for the real and imaginary parts of the relative electric permittivity ϵ_r and the relative magnetic permeability μ_r as a function of T_c with three different detunings ($\Delta/\gamma_{12} = 0.001, 0.002, 0.003$) are depicted in Figure 2. In the figure, we consider $N = 5 \times 10^{20} m^{-3}$, $|m_{32}/(d_{12}c)| \sim 10^{-2}$, $\gamma_{12} \sim 10^9 s^{-1}$ [18], $\gamma_{32} \sim 5 \times 10^{-4} \gamma_{12}$, $d_{12} \sim 50 Debye$ [19], $\omega_{31} = 0$ and $\delta = \Delta$. From Fig.2 (a) and (b), we can see that the curves of the real and imaginary part of ϵ_r are symmetrical when T_c varies from -0.5 to 0.5. Fig.2 (c) and (d) show that the real and imaginary parts of μ_r are almost symmetrical. For $\Delta = 0.003\gamma_{12}$ and $T_c = \pm 0.2\gamma_{12}$, the real parts of relative conductivity and permittivity are both negative. This means a DNIM is achieved. In other cases, $Re(\epsilon_r) < 0, Re(\mu_r) > 0$ or $Re(\epsilon_r) > 0, Re(\mu_r) < 0$, one can achieve a SNIM. We also demonstrate the influence of detuning on the relative conductivity and permittivity. As the detuning Δ increases from $0.001\gamma_{12}$ to $0.003\gamma_{12}$, the resonance peaks will enhance. Obviously, a small detuning is preferred in such a quantum dot system.

Figure 3 demonstrates the real and imaginary parts of the refractive index n and the ratio $-Re(n)/Im(n)$ as a function of the tunneling with $\Delta/\gamma_{12} = 0.001, 0.002, 0.003$. The other parameters are the same as Fig.2. For the real applications of the negative index materials, the ratio $-Re(n)/Im(n)$ has to be seriously considered since the low-loss negative index materials are desired. The curve of $Re(n)$ shows that the refractive index can be adjusted and controlled from -5 to 3 continuously for $\Delta = 0.001\gamma_{12}$. As T_c is changed in $-0.22\gamma_{12} < T_c < -0.03\gamma_{12}$ and $0.03\gamma_{12} < T_c < 0.22\gamma_{12}$, the real part of refractive index will be negative. The ratio $-Re(n)/Im(n)$ curve shows that the very low loss and the negative refractive index may be obtained at some point near $T_c = 0.18\gamma_{12}$ for $\Delta = 0.001\gamma_{12}$.

In conclusion, we have demonstrated the negative index of refraction can be obtained by means of voltage-controlled the coupled quantum dots system such as the vertically self-organized InAs quantum dots. As the bias voltage is changed, the refractive index can be adjusted continuously from negative to positive for some fixed detunings and simultaneously the loss of light in the system will be small. The SNIM and the DNIM can be achieved only by simply changing the external bias voltage. We hope that our prediction will be observed in the near future experiment.

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Fig. 1 (a) Schematic of the setup. A laser beam transmits from the upper layer to the lower layer. The substrates and the top surface are composed with transparent and conductive materials. (b) A typical vertically coupled InAs/InGaAs asymmetrical quantum dots system [12]. (c) The energy levels of a quantum dot system with the laser and the tunneling.

Fig. 2 The relative electric permittivity ϵ_r and the relative magnetic permeability μ_r as a function of T_c/γ_{12} with three different detunings ($\Delta/\gamma_{12} = 0.001, 0.002, 0.003$). The other parameters used are $N = 5 \times 10^{20} m^{-3}$, $\omega_{31} = 0$, and $\gamma_{32} = 5 \times 10^{-4} \gamma_{12}$.

Fig. 3 $Re(n)$ and $-Re(n)/Im(n)$ of the system with three detunings ($\Delta/\gamma_{12} = 0.001, 0.002, 0.003$). The other parameters are the same as in Fig.2.

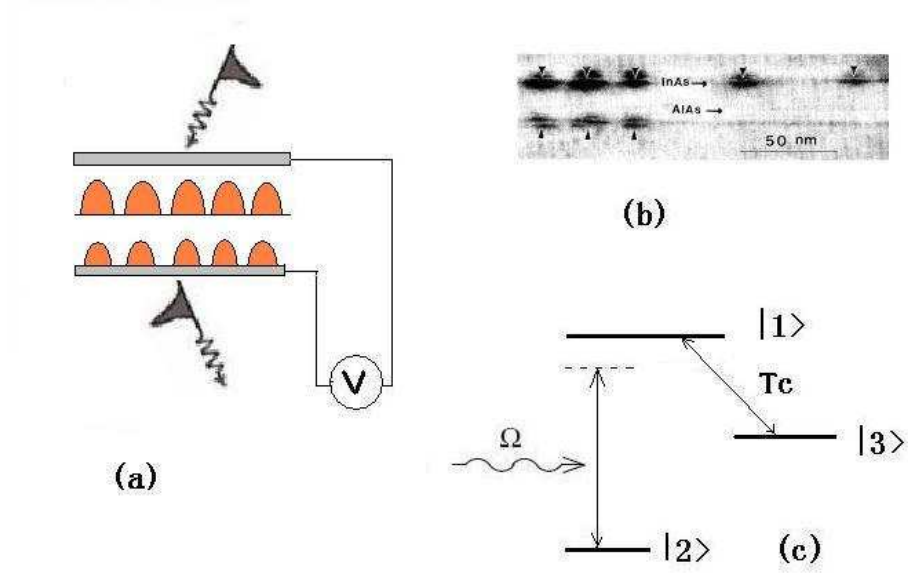


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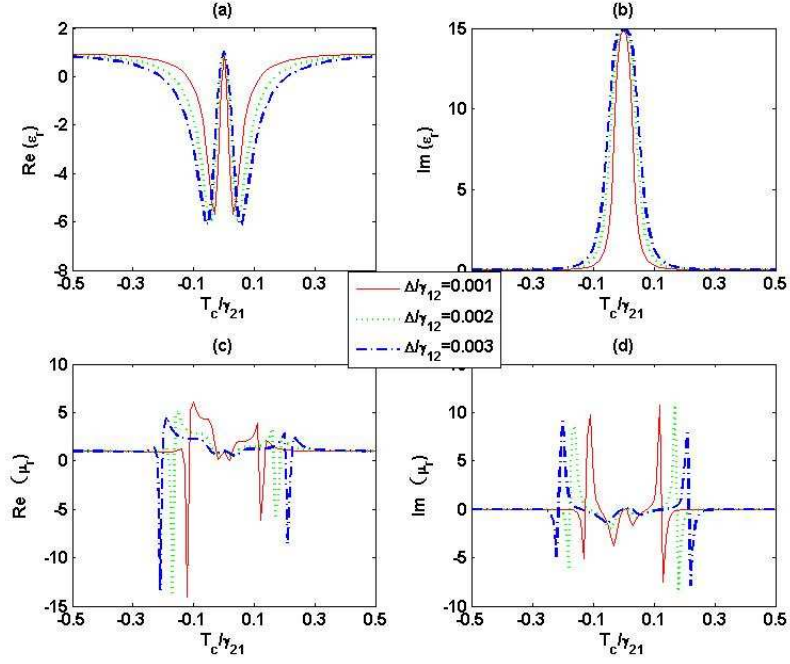


Fig. 2. The relative electric permittivity ϵ_r and the relative magnetic permeability μ_r as a function of T_c/γ_{12} with three different detunings ($\Delta/\gamma_{12} = 0.001, 0.002, 0.003$). The other parameters used are $N = 5 \times 10^{20} m^{-3}$, $\omega_{31} = 0$ and $\gamma_{32} = 5 \times 10^{-4} \gamma_{12}$.

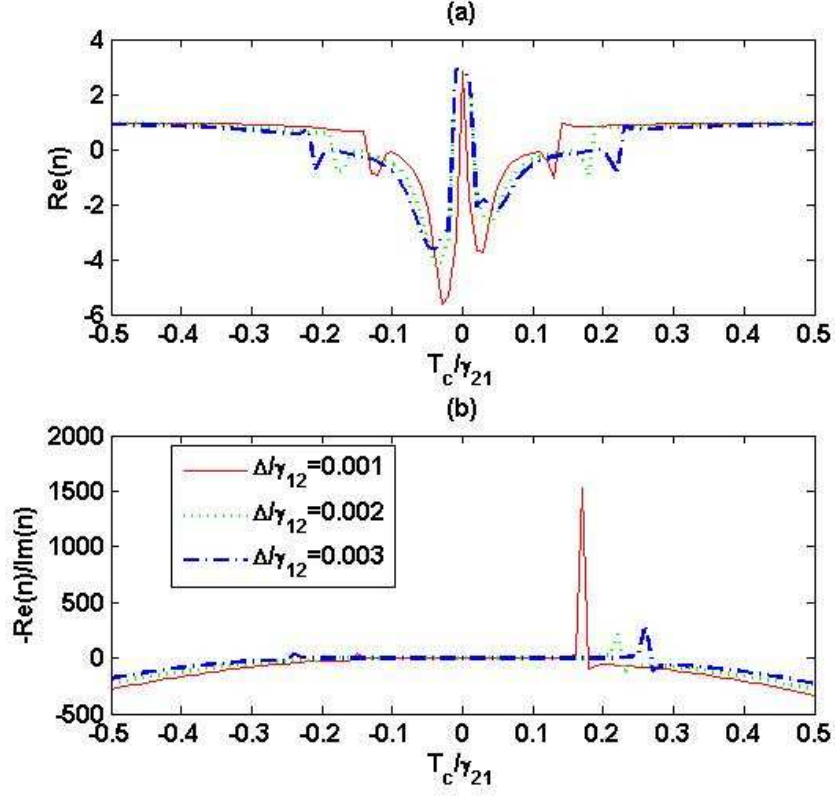


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